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Topic: Mixed Graph and Network Problems

Time: 45 mins Marks: /45 marks

No calculator allowed

Question One: [5, 5, 5: 15 marks]

For the graphs below,

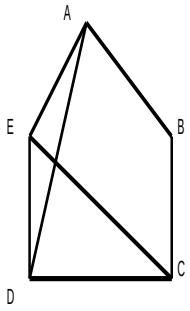
i) Define the set of vertices.

ii) State the number of edges

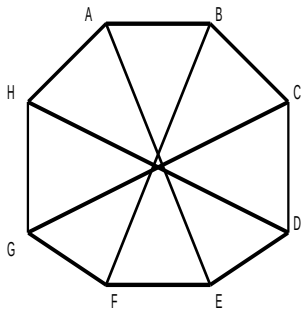
iii) State the number of regions

iv) Decide whether or not it is planar. If it is planar, redraw the graph without any intersecting edges.

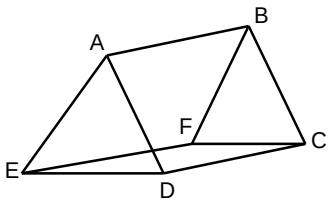
a)



b)



c)



Question Two: [2, 4, 4: 10 marks]

Five women are selected to take part in sampling some products. Mandy, Taylor, Shania, Lucy and Dion all enter a room and begin to greet each other. First Mandy shakes Taylor's hand, Shania shakes Lucy's hand and Dion waits. Then Dion shakes Mandy's hand. Taylor shakes Shania's hand and Lucy waits.

The product sampling session then starts before everyone has had a chance to shake each other's hands.

a) Draw a graph to represent the information above.

b) Draw the complement of the graph in a) and explain what it represents.

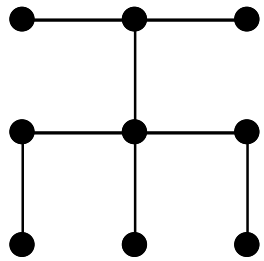
c) Draw a graph to represent all the introductions which need to be made and use it to calculate how many rounds it takes before everyone has shaken hands.

Question Three: [2, 2, 2, 2, 2: 10 marks]

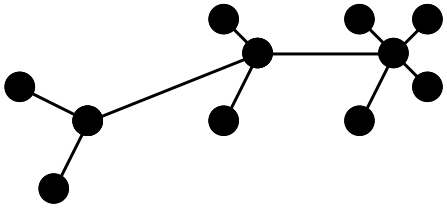
A tree graph is a planar graph with no cycles. It is an undirected graph in which any two vertices are connected by exactly one path. The following questions all refer to tree graphs.

a) For the following trees state the number of vertices and the number of edges.

i)



ii)



b) Draw a tree with 4 vertices and 3 edges.

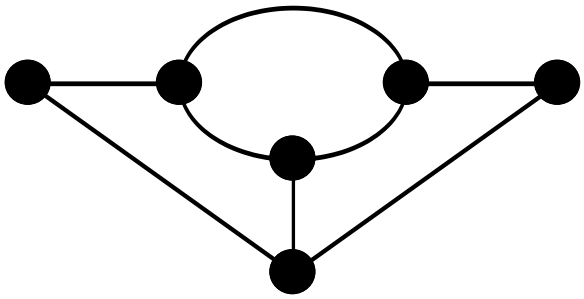
c) Is it possible to draw a tree with 4 vertices and 5 edges?

d) Use Euler's formula to justify why tree graphs are all planar.

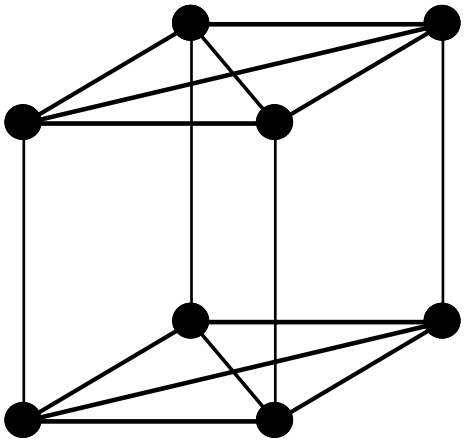
Question Four: [2, 2, 2: 6 marks]

For each of the following graphs, decide if it has an Eulerian cycle or a Eulerian walk, both or neither. Give a brief justification in each case.

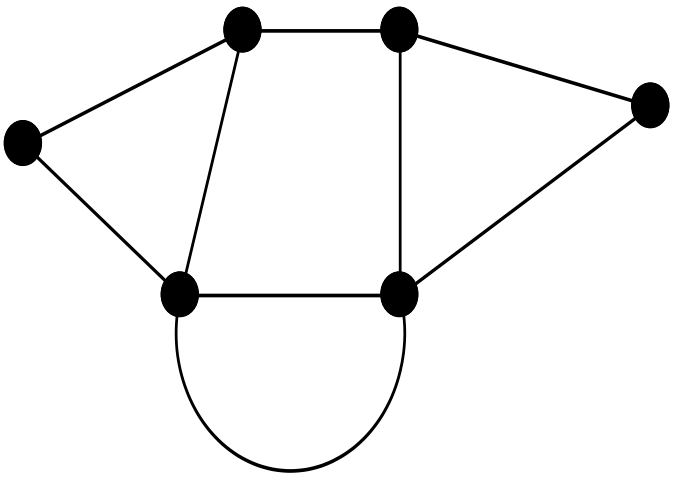
a)



b)

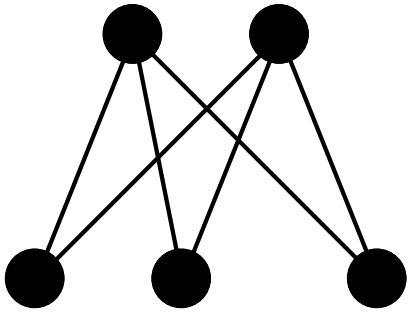


c)



Question Five: [2, 2: 4 marks]

Consider the following complete bipartite graph.



a) Give a brief justification as to why a Hamiltonian path exists in this network but not a Hamiltonian cycle.

b) What is the minimum number of extra edges you would need to add in order to make a Hamiltonian cycle possible? Add those paths to the graph above.

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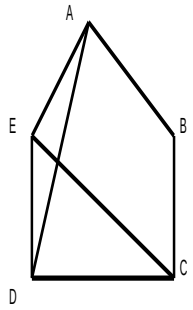
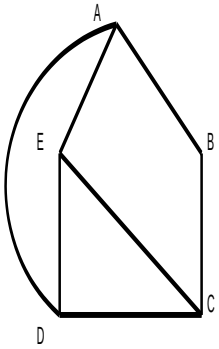
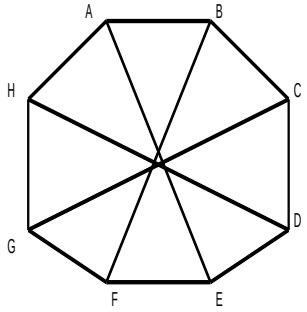
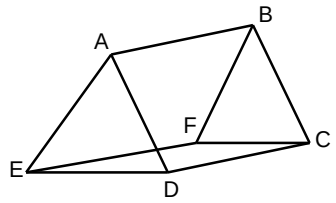
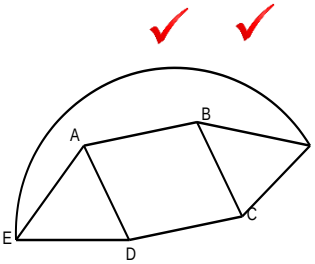
Topic: Mixed Graph and Network Problems SOLUTIONS

Time: 45 mins Marks: /45 marks

No calculator allowed

Question One: [5, 5, 5: 15 marks]

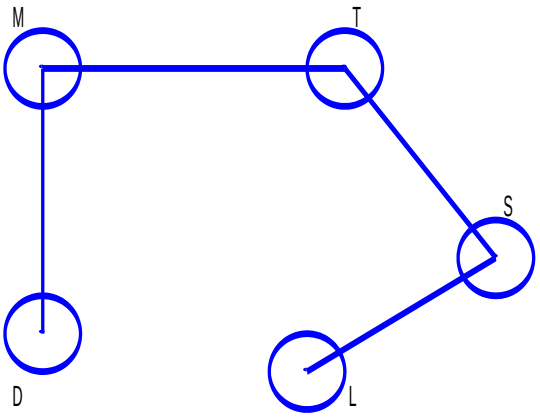
	Graph	Define set of vertices	Number of edges	Number of Regions	Is it planar? If yes redraw without any intersecting edges
1		$\{A, B, C, D, E\}$ ✓	7 ✓	4 ✓	Yes ✓ ✓ 
2		$\{A, B, C, D, E, F, G, H\}$ ✓	12 ✓	undefined ✓	No ✓ ✓
3		$\{A, B, C, D, E, F\}$ ✓	9 ✓	5 ✓	Yes ✓ ✓ 

Question Two: [2, 4, 4: 10 marks]

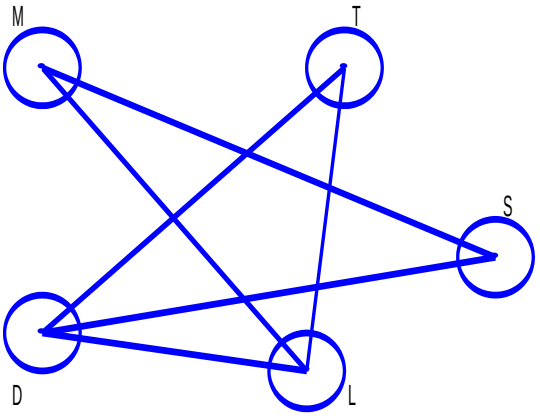
Five women are selected to take part in sampling some products. Mandy, Taylor, Shania, Lucy and Dion all enter a room and begin to greet each other. First Mandy shakes Taylor's hand, Shania shakes Lucy's hand and Dion waits. Then Dion shakes Mandy's hand. Taylor shakes Shania's hand and Lucy waits.

The product sampling session then starts before everyone has had a chance to shake each other's hands.

a) Draw a graph to represent the information above.

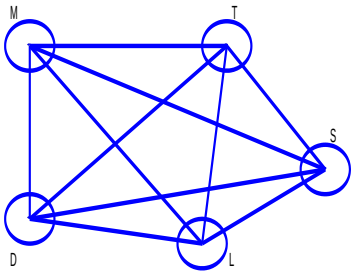


b) Draw the complement of the graph in a) and explain what it represents.



This graph represents all the handshakes which are yet to be made.

c) Draw a graph to represent all the introductions which need to be made and use it to calculate how many rounds it takes before everyone has shaken hands.



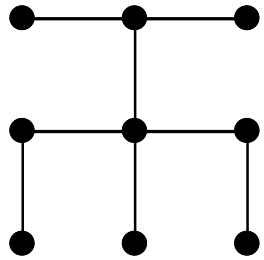
10 rounds of handshakes.

Question Three: [2, 2, 2, 2, 2: 10 marks]

A tree graph is a planar graph with no cycles. It is an undirected graph in which any two vertices are connected by exactly one path. The following questions all refer to tree graphs.

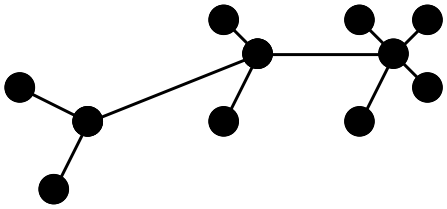
a) For the following trees state the number of vertices and the number of edges.

i)



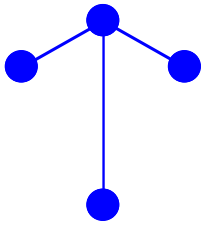
$V = 9$
$E = 8$ ✓ ✓

ii)



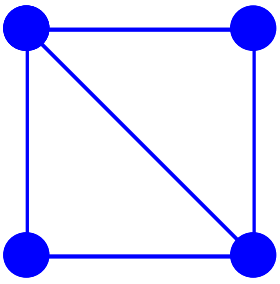
$V = 11$ $E = 10$ ✓ ✓

b) Draw a tree with 4 vertices and 3 edges.



✓ ✓

c) Is it possible to draw a tree with 4 vertices and 5 edges?



No, a tree must not be closed, it must only have 1 region and therefore a tree will always have more vertices then edges.

✓ ✓

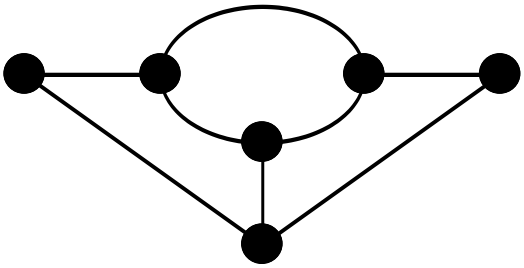
d) Use Euler's formula to justify why tree graphs are all planar.

Yes because the number of edges will always be one less than the number of vertices and since there are no cycles in tree graphs the region will always be 1. This will always satisfy Euler's formula thus making it planar. Let number of vertices be n, then number of edges is $n - 1$.
$n - (n - 1) + 2 = 1$ ✓ ✓

Question Four: [2, 2, 2: 6 marks]

For each of the following graphs, decide if it has an Eulerian cycle or a Eulerian walk, both or neither. Give a brief justification in each case.

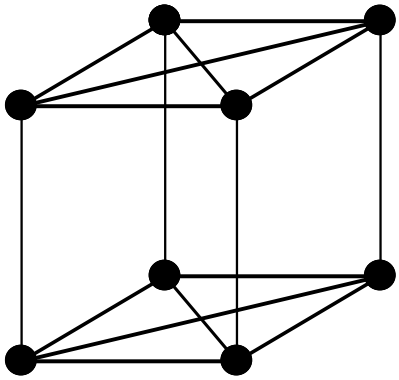
a)



✓ ✓

There are four odd vertices and therefore there is neither a Eulerian cycle or a Eulerian walk.

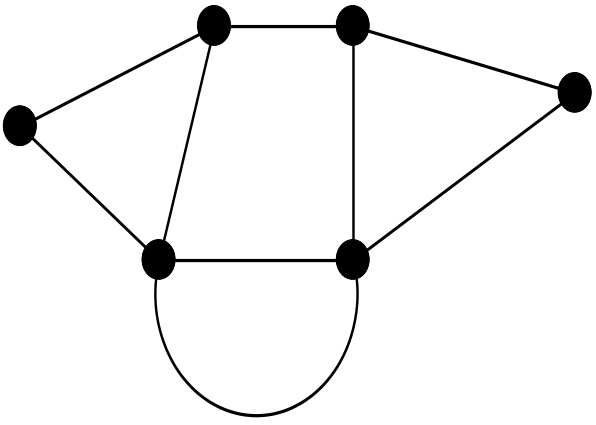
b)



All the vertices are even which means there is a Eulerian cycle.

✓ ✓

c)

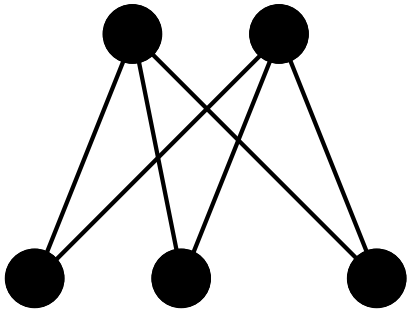


Two odd vertices mean that there is a Eulerian walk but it is not a cycle.

✓ ✓

Question Five: [2, 2: 4 marks]

Consider the following complete bipartite graph.



a) Give a brief justification as to why a Hamiltonian path exists in this network but not a Hamiltonian cycle.

✓

A Hamiltonian walk visits each vertex once. In this graph since none of the horizontally adjacent vertices are connected the Hamiltonian path must (if starting at the bottom left) go up then down then up then down. We are finishing on the bottom right and have no way of connecting back to the bottom left where we started.

✓

b) What is the minimum number of extra edges you would need to add in order to make a Hamiltonian cycle possible? Add those paths to the graph above and show the cycle.

1 path is required.

✓